On the Determination of the Distances of a Comet from the Earth from Three Observations. By Dr. F. W. Berg.

Let us designate by $K_{\prime\prime}$ the longitude of the ascending node, and by $T_{\prime\prime}$ the inclination to the ecliptic, of a great circle passing through the first and second observed places of the heavenly body, which may be given by means of $\alpha, \alpha', \beta, \beta'$, and we have—

$$\tan\beta = \sin(\alpha - K_{\prime\prime}) \tan T_{\prime\prime},$$
$$\tan\beta' = \sin(\alpha' - K_{\prime\prime}) \tan T_{\prime\prime};$$

and therefore, for the great circle passing through the second and third places, and for the great circle passing through the first and third places, we obtain—

$$\tan\beta' = \sin(\alpha' - K\ ) \tan T,$$
$$\tan\beta'' = \sin(\alpha'' - K\ ) \tan T,$$
$$\tan\beta = \sin(\alpha - K_{\prime}) \tan T_{\prime},$$
$$\tan\beta'' = \sin(\alpha'' - K_{\prime\prime}) \tan T_{\prime\prime}.$$

Substituting these abbreviations in the Gaussian expressions (O.1.2.), (o.O.2), etc. (conf. *Theoria Motus*, art. 113), we find—

$$(\mathrm{O.1.2}) = \sin(\alpha'' - \alpha').\mathrm{M}, \quad (\mathrm{I.1.2}) = \sin(\alpha'' - \alpha').\mathrm{M}', \quad (\mathrm{II.1.2}) = \sin(\alpha'' - \alpha').\mathrm{M}'',$$
$$(\mathrm{o.O.2}) = \sin(\alpha - \alpha'').\mathrm{M}_{\prime}, \quad (\mathrm{o.I.2}) = \sin(\alpha - \alpha'').\mathrm{M}_{\prime}', \quad (\mathrm{o.II.2}) = \sin(\alpha - \alpha'').\mathrm{M}_{\prime}'',$$
$$(\mathrm{o.1.O}) = \sin(\alpha' - \alpha).\mathrm{M}_{\prime\prime}, \quad (\mathrm{o.1.I}) = \sin(\alpha' - \alpha).\mathrm{M}_{\prime\prime}', \quad (\mathrm{o.1.II}) = \sin(\alpha' - \alpha).\mathrm{M}_{\prime\prime}'',$$

if we put—

$$\mathrm{M} = \tan B + \sin(K - L) \tan T,$$
$$\mathrm{M}_{\prime} = \tan B + \sin(K_{\prime} - L) \tan T_{\prime},$$
$$\mathrm{M}_{\prime\prime} = \tan B + \sin(K_{\prime\prime} - L) \tan T_{\prime\prime};$$

and these expressions will give $M', M_{\prime}', M_{\prime\prime}', M'', M_{\prime}'', M_{\prime\prime}''$, if we change B and L into B' L' and B'' L''.

Therefore, we obtain from the equations (9), (10), (11) (conf. *Theoria Motus*, art. 114)—

$$\text{(A)} \qquad \begin{aligned} 0 &= n\ (\mathrm{o.1.2})\ \delta + \sin(\alpha'' - \alpha').\mathrm{P}, \\ 0 &= n'\ (\mathrm{o.1.2})\ \delta' + \sin(\alpha'' - \alpha).\mathrm{P}_{\prime}, \\ 0 &= n''\ (\mathrm{o.1.2})\ \delta'' + \sin(\alpha' - \alpha).\mathrm{P}_{\prime\prime}, \end{aligned}$$

if we put

$$\mathrm{P} = n\mathrm{M}\ \mathrm{D} - n'\mathrm{M}'\ \mathrm{D}' + n''\mathrm{M}''\ \mathrm{D}'',$$
$$\mathrm{P}_{\prime} = n\mathrm{M}_{\prime}\mathrm{D} - n'\mathrm{M}_{\prime}'\mathrm{D}' + n''\mathrm{M}_{\prime}''\mathrm{D}'',$$
$$\mathrm{P}_{\prime\prime} = n\mathrm{M}_{\prime\prime}\mathrm{D} - n'\mathrm{M}_{\prime\prime}'\mathrm{D}' + n''\mathrm{M}_{\prime\prime}''\mathrm{D}''.$$

For the case where the great circle through the first and third observed places of the body passes also through the second place, we have

$$(\mathrm{o.1.2}) = 0,$$

and

$$T = T_{\prime} = T_{\prime\prime}, \qquad K = K_{\prime} = K_{\prime\prime};$$

and, consequently,

$$M = M_{,} = M_{,,},$$
$$M' = M_{,}' = M_{,,}',$$
$$M'' = M_{,}'' = M_{,,}'',$$

and the equations (A) give only the relation

$$P = P_{,} = P_{,,} = 0,$$

whence

(B) $$nMD - n'M'D' + n''M''D'' = 0.$$

It is to be here remarked that the values $P\ P_{,}\ P_{,,}$ are of the third order with reference to the intervals of time.

In the formula (B) we may change n, n', n'' into N, N', N'', where N, N', N'' have the same signification for the orbit of the Earth which have n, n', n'' for the orbit of the comet.

We hence obtain

(C) $$NMD - N'M'D' + N''M''D'' = 0;$$

and, consequently, we obtain from (B) and (C)

$$\left(\frac{n}{n'} - \frac{N}{N'}\right) MD + \left(\frac{n''}{n'} - \frac{N''}{N'}\right) M''D'' = 0;$$

and therefore

(D) $$n : n' : n'' = N : N' : N''.^*$$

If we divide the third of the equations (A) by the first, we obtain in general:

(E) $$\frac{\delta''}{\delta} = \frac{n}{n''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha')} \cdot \frac{P_{,,}}{P};$$

and, consequently, in the case in which

$$(0.1.2) = 0,$$

we obtain

(F) $$\frac{\delta''}{\delta} = \frac{n}{n''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha')};$$

or, having regard to (D),

(G) $$\frac{\delta''}{\delta} = \frac{N}{N''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha')}.$$

The value of the fraction $\frac{P_{,,}}{P}$ is equal to unity only when the equation $(0.1.2) = 0$ is accurately true; but if the equation $(0.1.2)$

* Compare Gauss, *Theoria Motus Corporum Cœlestium*, art. 162, and Olbers, *Abhandlung über die leichteste Methode die Bahn eines Cometes zu bestimmenm, herausg. von Encke*, Weimar, 1847, p. 80.

$=0$ is only the result of errors of observation, the value of the fraction $\frac{P_{\prime\prime}}{P}$ may be affected with an error of the order zero with reference to the intervals of time, which error may be sometimes large.*

Wilna Observatory,
April 27 (*May* 9), 1876.

Note by the Editor.—In the foregoing paper the notation is as follows:

α, geocentric longitude of Comet,

β, „ latitude,

δ, „ distance, projected on plane of Ecliptic,

L, heliocentric longitude of Earth,

B, „ latitude,

D, „ distance, projected on plane of Ecliptic,

viz., these values refer to the first observation; the corresponding accented quantities α' etc. refer to the second observation, and the doubly accented quantities α'' etc. to the third observation.

We have for the great circle passing through the second and third geocentric places of the Comet:

K, longitude of ascending node,

T, inclination,

in reference to the Ecliptic; and similarly for great circle through third and first places $K_{\prime}$, $T_{\prime}$; and for great circle through first and second places $K_{\prime\prime}$, $T_{\prime\prime}$.

The Gaussian function (0, 1, 2) denotes

$$\tan\beta\sin(\alpha''-\alpha'') + \tan\beta'\sin(\alpha-\alpha'') + \tan\beta''\sin(\alpha'-\alpha);$$

and if herein for (α, β), (α', β'), or (α'', β''), we write (L, B) (L′, B′) or (L″, B″) the symbol 0, 1, or 2, as the case may be, is changed into O, I, or II.

Errata.

Page 306, second line from bottom, before the words "in the Dorpat," the words "*not seen*" should be inserted.

Page 307, seventeenth line from bottom, after the words "which shew" the word "*no*" should be inserted.

These errors reverse the meaning.

Page 308, fifth line from top, for *speculæ* read *specula*.

* Compare *Briefwechsel zwischen Olbers und Bessel, herausg. von Erman*, Leipzig, 1852, Band I. pp. 384 and 387.